

# **Relationship to Thermodynamics**

**Chapter One**

**Section 1.3**

# CONSERVATION OF ENERGY (FIRST LAW OF THERMODYNAMICS)

- An important tool in heat transfer analysis, often providing the **basis for determining** the **temperature** of a system.
- Alternative Formulations

Time Basis:

At an instant

or

Over a time interval

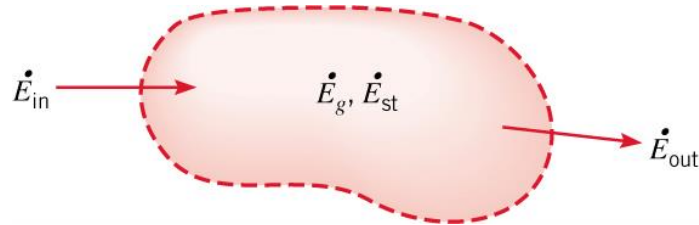
Type of System:

Control volume

Control surface

# APPLICATION TO A CONTROL VOLUME

- At an **Instant of Time**:



Note representation of system by a **control surface (dashed line)** at the boundaries.

## Surface Phenomena

$\dot{E}_{in}, \dot{E}_{out}$  : rate of thermal and/or mechanical **energy transfer across the control surface** due to heat transfer, fluid flow and/or work interactions.

## Volumetric Phenomena

$\dot{E}_g$  : rate of **thermal energy generation** due to conversion from another energy form (e.g., electrical, nuclear, or chemical); energy conversion process occurs within the system.

$\dot{E}_{st}$  : rate of change of **energy storage in the system**.

## Conservation of Energy

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \frac{dE_{st}}{dt} \equiv \dot{E}_{st} \quad (1.12c)$$

Each term has units of J/s or W.

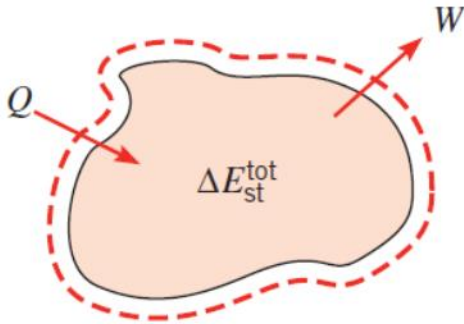
- Over a **Time Interval**

$$E_{in} - E_{out} + E_g = \Delta E_{st} \quad (1.12b)$$

Each term has units of J.

- Special Cases (Linkages to Thermodynamics)

(i) **Transient** Process for a **Closed System** of Mass ( $m$ ) Assuming Heat Transfer to the System (Inflow) and Work Done by the System (Outflow).



Over a **time interval**

$$Q - W = \Delta E_{st}^{\text{tot}} \quad (1.12a)$$

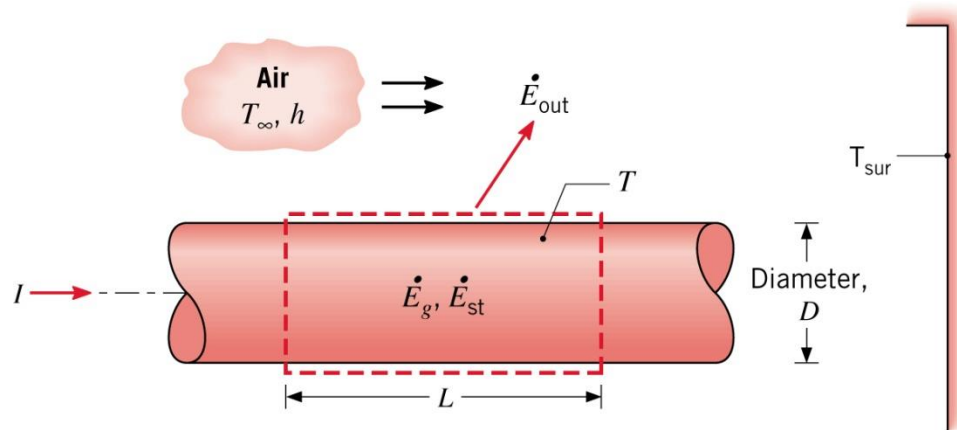
For negligible changes in potential or kinetic energy

$$Q - W = \Delta U_t$$

At an **instant**

$$q - \dot{W} = \frac{dU_t}{dt}$$

### Example 1.4: Application to thermal response of a conductor with Ohmic heating (generation):

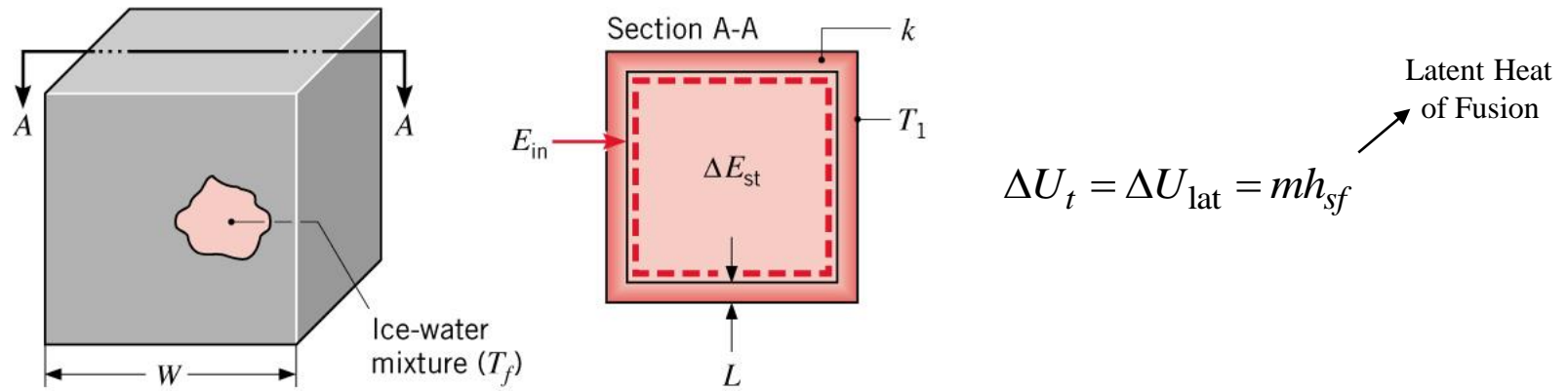


- Involves change in **thermal energy** and for an incompressible substance.

$$\frac{dU_t}{dt} = mc \frac{dT}{dt} \quad \text{💬}$$

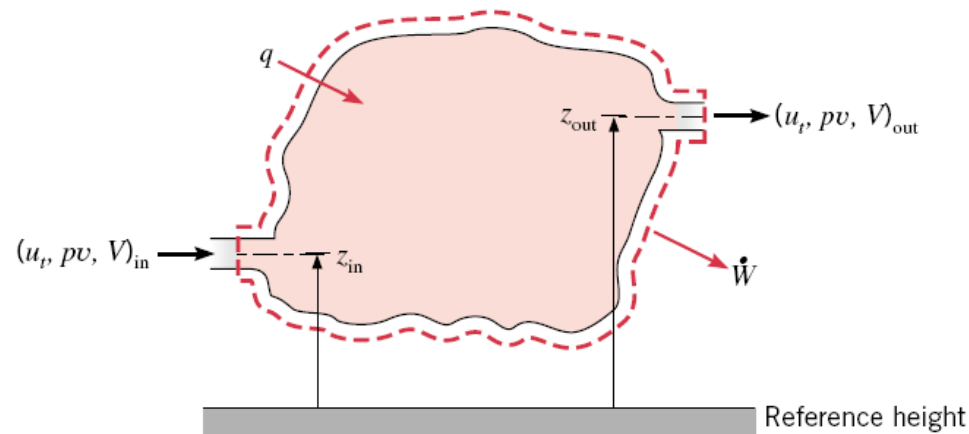
- Heat transfer is from the conductor (negative  $q$ )
- Generation may be viewed as **electrical work** done on the system (negative  $\dot{W}$ )

Example 1.5: Application to isothermal solid-liquid phase change in a container:



## Open System

(ii) **Steady State** for Flow through an **Open System** without Phase Change or Generation:



At an **Instant of Time**:

$$\dot{m} \left( u_t + pv + V^2/2 + gz \right)_{in} + q - \dot{m} \left( u_t + pv + V^2/2 + gz \right)_{out} - \dot{W} = 0 \quad (1.12d)$$

- $(pv) \rightarrow$  **flow work**
- $(u_t + pv) \equiv i \rightarrow$  **enthalpy**
- For an **ideal gas** with **constant specific heat**:

$$i_{in} - i_{out} = c_p (T_{in} - T_{out})$$

- For an **incompressible liquid**:

$$u_{in} - u_{out} = c (T_{in} - T_{out})$$

$$(pv)_{in} - (pv)_{out} \approx 0$$

- For systems with significant heat transfer:

$$\left( V^2/2 \right)_{in} - \left( V^2/2 \right)_{out} \approx 0$$

$$(gz)_{in} - (gz)_{out} \approx 0$$

# THE SURFACE ENERGY BALANCE

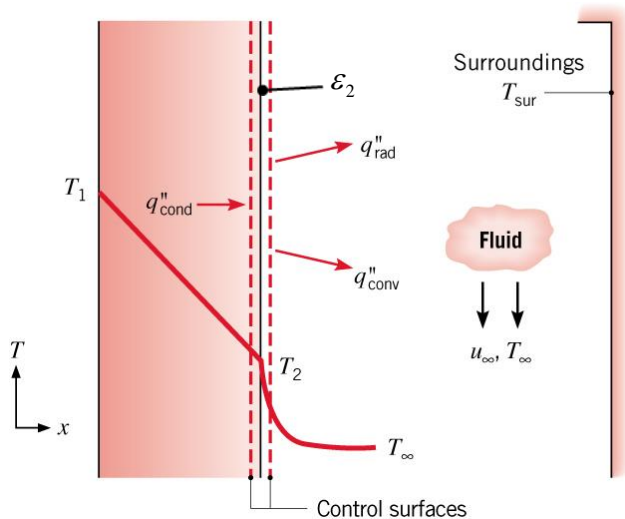
A special case for which **no volume or mass** is encompassed by the control surface.

**Conservation of Energy** (Instant in Time):

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0 \quad \text{☰} \quad (1.13)$$

- With no mass and volume, energy storage and generation are not pertinent to the energy balance, even if they occur in the medium bounded by the surface. (Applies for steady-state and transient conditions.)

Consider surface of wall with heat transfer by conduction, convection and radiation.





$$q''_{\text{cond}} - q''_{\text{conv}} - q''_{\text{rad}} = 0$$

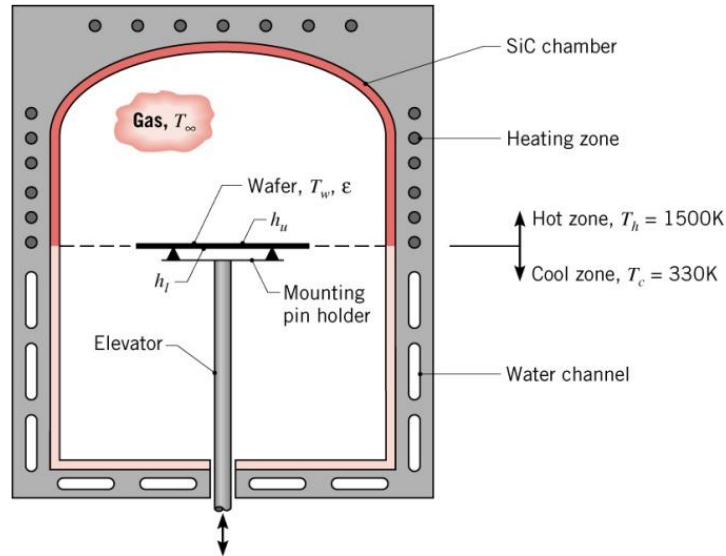
$$k \frac{T_1 - T_2}{L} - h(T_2 - T_\infty) - \epsilon_2 \sigma (T_2^4 - T_{\text{sur}}^4) = 0$$



# METHODOLOGY OF FIRST LAW ANALYSIS

- On a **schematic** of the system, represent the **control surface** by dashed line(s).
- Choose the appropriate **time basis**. 
- **Identify relevant energy** transport, generation and/or storage **terms** by **labeled arrows** on the schematic.
- Write the governing form of the **Conservation of Energy** requirement.
- Substitute appropriate expressions for terms of the energy equation.
- Solve for the unknown quantity. 

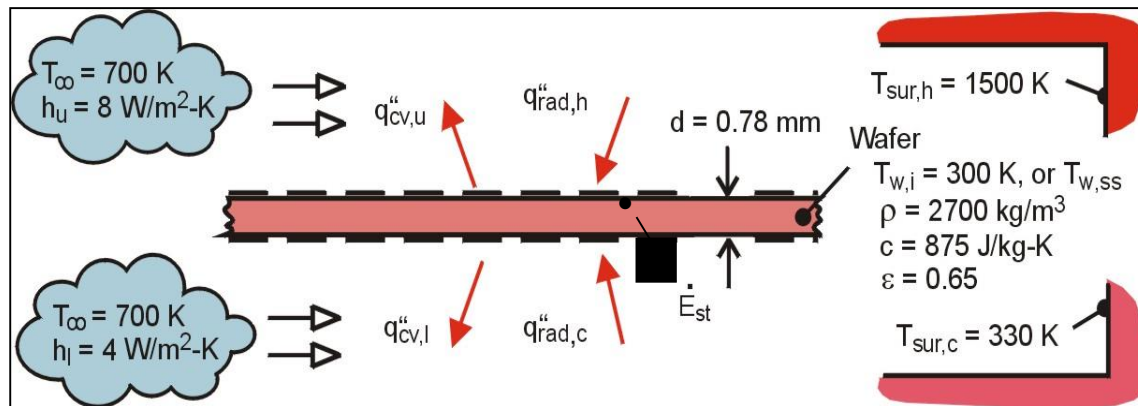
**Problem 1.46:** Thermal processing of silicon wafers in a two-zone furnace. Determine (a) the initial rate of change of the wafer temperature and (b) the steady-state temperature.



**KNOWN:** Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

**FIND:** (a) Initial rate of change of the wafer temperature from a value of  $T_{w,i} = 300$  K, and (b) steady-state temperature. Is convection significant? Sketch the variation of wafer temperature with vertical distance.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Wafer temperature is uniform, (2) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), (4)  $\alpha = \varepsilon$ , and (5) Negligible heat losses from wafer to pin holder.

**ANALYSIS:** The energy balance on the wafer includes convection to the upper (u) and lower (l) surfaces from the ambient gas, radiation exchange with the hot- and cool-zones and an energy storage term for the transient condition. Hence, from Eq. (1.12c),

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$$

or, per unit surface area

$$q''_{\text{rad},h} + q''_{\text{rad},c} - q''_{\text{cv},u} - q''_{\text{cv},l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon \sigma (T_{\text{sur},h}^4 - T_w^4) + \varepsilon \sigma (T_{\text{sur},c}^4 - T_w^4) - h_u (T_w - T_\infty) - h_l (T_w - T_\infty) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of change of the wafer temperature is determined using the foregoing energy balance with  $T_w = T_{w,i} = 300 \text{ K}$ ,

$$0.65 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (1500^4 - 300^4) \text{ K}^4 + 0.65 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (330^4 - 300^4) \text{ K}^4$$

$$- 8 \text{ W/m}^2 \cdot \text{K} (300 - 700) \text{ K} - 4 \text{ W/m}^2 \cdot \text{K} (300 - 700) \text{ K} =$$

$$2700 \text{ kg/m}^3 \times 875 \text{ J/kg} \cdot \text{K} \times 0.00078 \text{ m} (dT_w/dt)_i$$

$$(dT_w/dt)_i = 104 \text{ K/s}$$

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Problem: Silicon Wafer (cont.)

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature,  $T_w = T_{w,ss}$ .

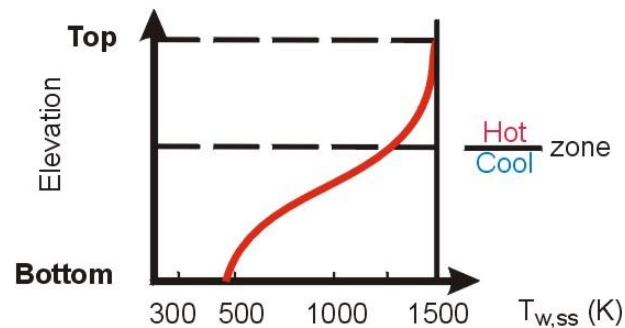
$$0.65\sigma(1500^4 - T_{w,ss}^4)K^4 + 0.65\sigma(330^4 - T_{w,ss}^4)K^4 - 8W/m^2 \cdot K(T_{w,ss} - 700)K - 4W/m^2 \cdot K(T_{w,ss} - 700)K = 0$$

$$T_{w,ss} = 1251 K$$

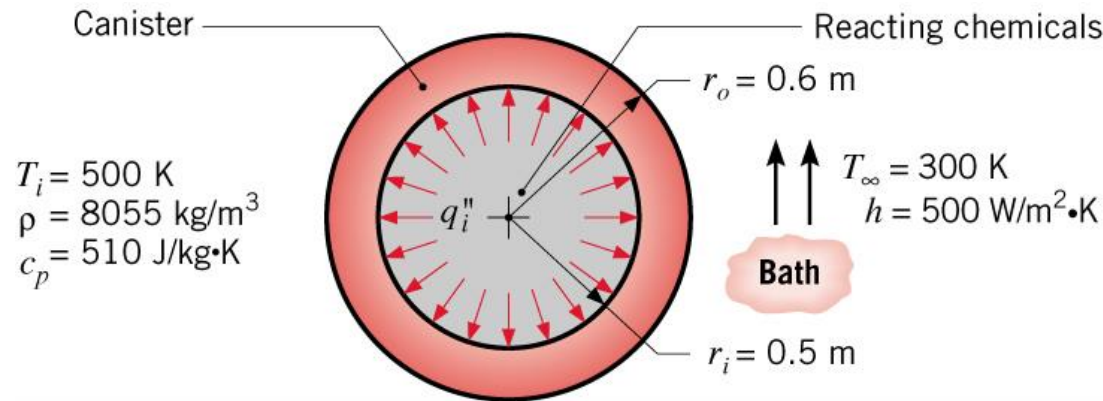
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To assess the relative importance of convection, solve the energy balances assuming no convection. With  $(dT_w/dt)_i = 101 K/s$  and  $T_{w,ss} = 1262 K$ , we conclude that the radiation exchange processes control the initial rate of change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone, and its temperature would decrease. The temperature-distance relation might appear as shown in the sketch.



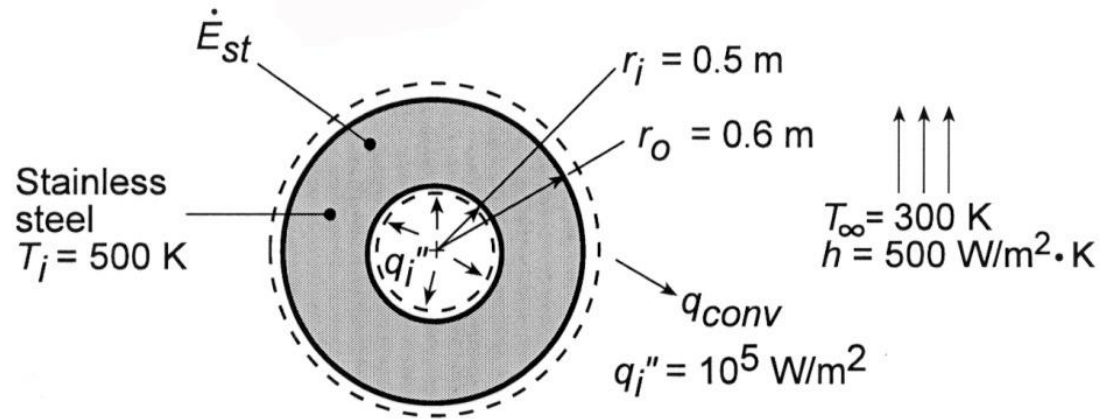
**Problem 1.50:** Cooling of spherical canister used to store reacting chemicals. Determine (a) the initial rate of change of the canister temperature, (b) the steady-state temperature, and (c) the effect of convection on the steady-state temperature.



**KNOWN:** Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

**FIND:** (a) Governing equation for variation of wall temperature with time and the initial rate of change, (b) Steady-state wall temperature and, (c) Effect of convection coefficient on canister temperature.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

**PROPERTIES:** *Table A.1*, Stainless Steel, AISI 302:  $\rho = 8055 \text{ kg/m}^3$ ,  $c_p = 510 \text{ J/kg}\cdot\text{K}$ .

**ANALYSIS:** (a) Performing an energy balance on the shell at an instant of time,  $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$ . Identifying relevant processes and solving for  $dT/dt$ ,

$$q_i''(4\pi r_i^2) - h(4\pi r_o^2)(T - T_\infty) = \rho \frac{4}{3} \pi (r_o^3 - r_i^3) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p (r_o^3 - r_i^3)} \left[ q_i'' r_i^2 - h r_o^2 (T - T_\infty) \right]$$

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Substituting numerical values for the initial condition, find

$$\left. \frac{dT}{dt} \right|_i = \frac{3 \left[ 10^5 \frac{\text{W}}{\text{m}^2} (0.5\text{m})^2 - 500 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (0.6\text{m})^2 (500 - 300) \text{K} \right]}{8055 \frac{\text{kg}}{\text{m}^3} 510 \frac{\text{J}}{\text{kg} \cdot \text{K}} \left[ (0.6)^3 - (0.5)^3 \right] \text{m}^3}$$

$$\left. \frac{dT}{dt} \right|_i = -0.084 \text{ K/s}$$

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(b) Under steady-state conditions with  $\dot{E}_{\text{st}} = 0$ , it follows that

$$q_i'' (4\pi r_i^2) = h (4\pi r_o^2) (T - T_\infty)$$

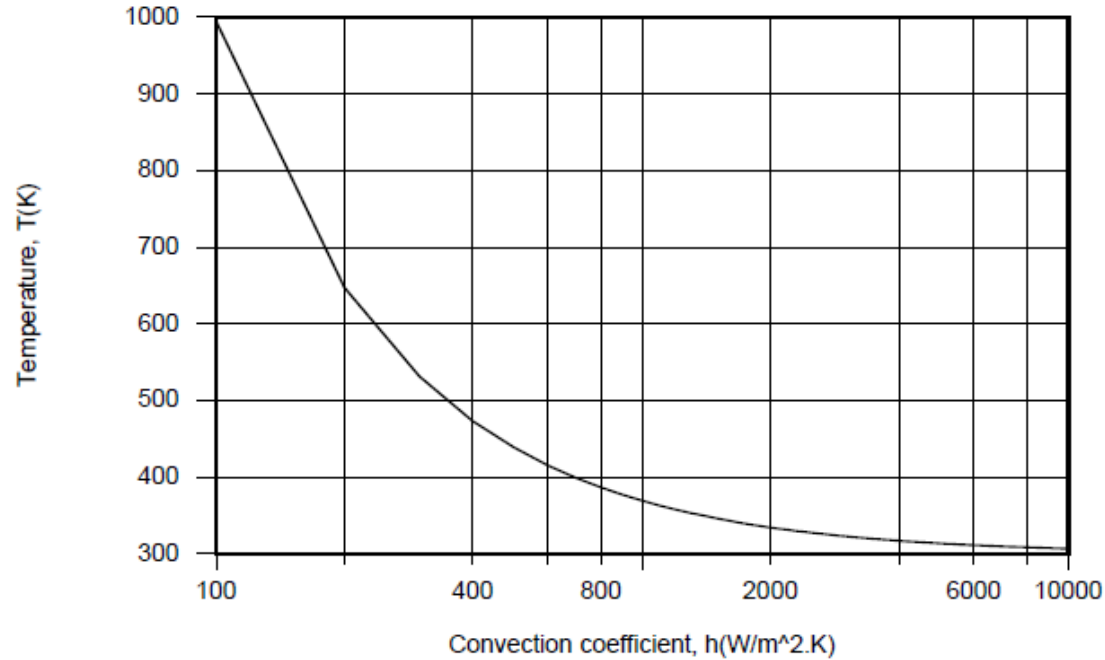
$$T = T_\infty + \frac{q_i''}{h} \left( \frac{r_i}{r_o} \right)^2 = 300\text{K} + \frac{10^5 \text{W/m}^2}{500 \text{W/m}^2 \cdot \text{K}} \left( \frac{0.5\text{m}}{0.6\text{m}} \right)^2 = 439\text{K}$$

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(c) Parametric calculations show a sharp increase in temperature with decreasing values of  $h < 1000 \text{ W/m}^2 \cdot \text{K}$ . For  $T > 380 \text{ K}$ , boiling will occur at the canister surface, and for  $T > 410 \text{ K}$  a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in  $h$  and increase in  $T$ .



Problem: Cooling of Spherical Canister (cont.)



Although the canister remains well below the melting point of stainless steel for  $h = 100 \text{ W/m}^2\cdot\text{K}$ , boiling should be avoided, in which case the convection coefficient should be maintained at  $h > 1000 \text{ W/m}^2\cdot\text{K}$ .

**COMMENTS:** The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is  $\theta = (S/R)(1 - e^{-Rt}) + \theta_i e^{-Rt}$ , where  $\theta \equiv T - T_\infty$ ,

$S \equiv 3q_i'' r_i^2 / \rho c_p (r_o^3 - r_i^3)$ ,  $R = 3hr_o^2 / \rho c_p (r_o^3 - r_i^3)$ . Note results for  $t \rightarrow \infty$  and for  $S = 0$ .



# SECOND LAW OF THERMODYNAMICS

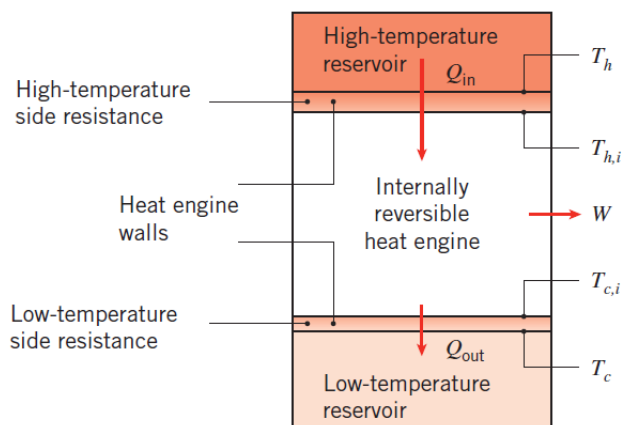
- An important tool to determine how heat transfer affects the **efficiency of energy conversion**.

For a reversible heat engine **neglecting heat transfer effects** between the heat engine and large reservoirs, the **Carnot efficiency** is

$$\eta_C = \frac{W}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{T_c}{T_h} \quad (1.15, 1.16)$$

where  $T_c$  and  $T_h$  are the absolute temperatures of large cold and hot reservoirs, respectively.

For an **internally reversible** heat engine **with** heat transfer to and from the large reservoirs properly accounted for, the **modified Carnot efficiency** is



$$\eta_m = \frac{W}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_{c,i}}{T_{h,i}} \quad (1.17)$$

where  $T_{c,i} > T_c$  and  $T_{h,i} < T_h$  are the absolute temperatures **seen by the internally reversible heat engine**. Note that  $q_{\text{out}}$  and  $q_{\text{in}}$  are heat transfer rates (J/s or W).

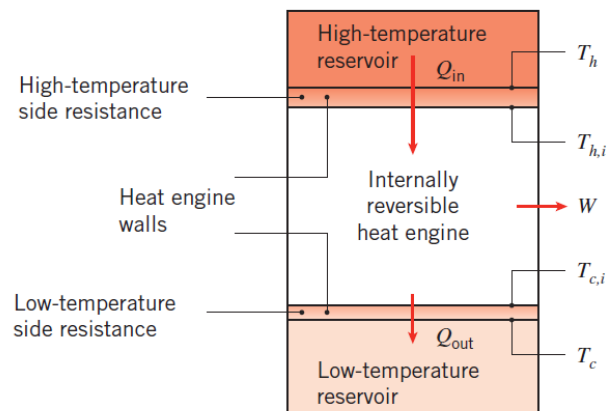
- **Heat transfer resistances** associated with, for example, walls separating the internally reversible heat engine from the hot and cold reservoirs relate the heat transfer rates to temperature differences:

$$(T_h - T_{h,i}) = q_{in} R_{t,h} \quad (T_{c,i} - T_c) = q_{out} R_{t,c} \quad (1.18 \text{ a,b})$$

In reality, heat transfer resistances (K/W) must be non-zero since according to the rate equations, for any temperature difference only a finite amount of heat may be transferred.

The modified Carnot efficiency may ultimately be expressed as

$$\eta_m = 1 - \frac{T_c}{T_h - q_{in} R_{tot}} \quad \text{where } R_{tot} = R_{t,h} + R_{t,c} \quad (1.21)$$



From Eq. 1.21,

- $\eta_m = \eta_C$  only if  $R_{tot}$  could be made infinitely small.
- For **realistic** situations ( $R_{tot} \neq 0$ ),  $\eta_m < \eta_C$ .
- Good heat transfer engineering is a key to improve the efficiency of heat engines.